

Graph-constrained Diffusion For End-to-end Path Planning

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Highlights

1. We devise the **first diffusion process in graph space**
2. We propose an **end-to-end path planning** method
3. Our methods achieves **state-of-the-art** performances

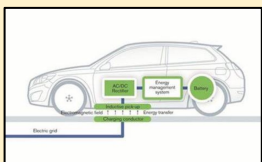
Motivation

Limitations of traditional search-based path planning

1. Many factors can hardly be modeled in closed form.
2. The paths' cost is assumed as the summation of edges, which does not always hold.

User option: detour due to implicit factors
eg. safety/leisure/...

Shortest
Fastest



Non-linearity of discharging rate breaks linear accumulation assumption

Our Goal:

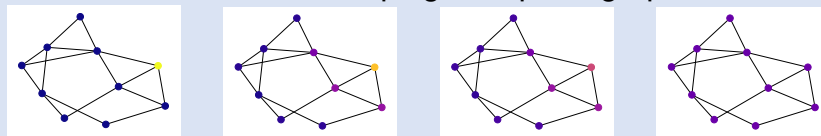
1. Capture path distribution by powerful diffusion model
2. Convert planning into conditional sampling, bypassing graph search.

Methods

1. Diffusion for a single vertex

$$\text{Forward } q(v_t|v_{t-1}) = \text{Cat}(v_t|\mathbf{p} = q(v_{t-1})\mathbf{Q}_t) \quad \frac{\partial \mathbf{h}}{\partial t} = \Delta \mathbf{h}$$

$$\mathbf{C}_t = \exp\{(\mathbf{A} - \mathbf{D})t\} \quad \text{Derived from heat conduction, hoping to capture graph structure}$$



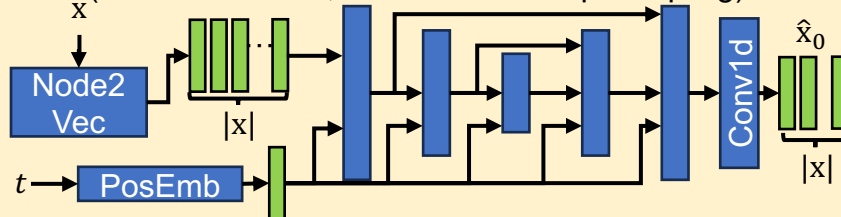
$$\text{Reverse } q(v_{t-1}|v_t, v_0) = \text{Cat}(v_{t-1}|\mathbf{p} \propto v_t \mathbf{C}_t \odot v_0 \bar{\mathbf{C}}_{t-1})$$

2. Diffusion for paths

$$\text{Forward: } q(\mathbf{x}_t|\mathbf{x}_0) = \prod_{i=1}^{|\mathbf{x}|} \text{Cat}(\mathbf{x}_t^i|\mathbf{p} = \mathbf{x}_0^i \bar{\mathbf{C}}_t)$$

$$\text{Reverse: } q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0) = \prod_{i=1}^{|\mathbf{x}|} \text{Cat}(\mathbf{x}_{t-1}^i|\mathbf{p} \propto \mathbf{x}_t^i \mathbf{C}_t \odot \mathbf{x}_0^i \bar{\mathbf{C}}_{t-1})$$

We adopt a U-net structure for estimating \mathbf{x}_0
(1-D Convolution, Remove Down/up sampling)

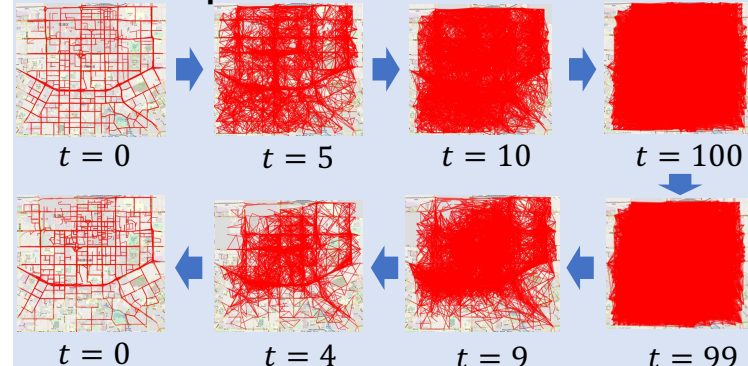


3. Add prior evidence taking origin/destination as condition

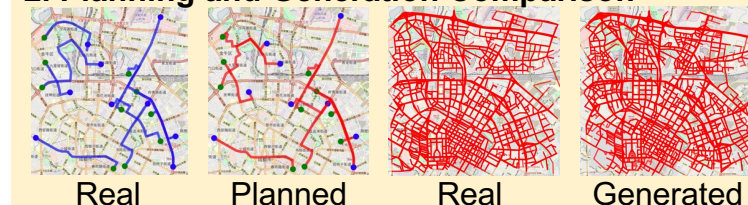
$$\tilde{\mathbf{p}}(\mathbf{x}) = \mathbf{p}_\theta(\mathbf{x}) \mathbf{h}(\mathbf{x}|\text{ori}, \text{dst}) \quad \text{Adopt a MHA structure}$$

Experiments

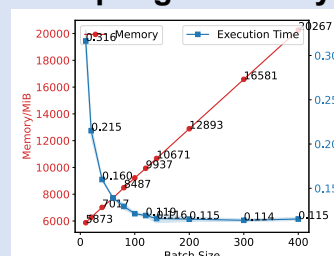
1. Diffusion process illustration



2. Planning and Generation Comparison



3. Sampling Efficiency



4. Hit Ratio

